

# *fastppm*: fast tumor phylogeny regression via tree-structured dual dynamic programming

Henri Schmidt<sup>\*,1</sup>, Yuanyuan Qi<sup>\*,2</sup>, Ben Raphael<sup>1</sup>, and Mohammed El-Kebir<sup>2</sup>

\* Denotes equal contribution.

1

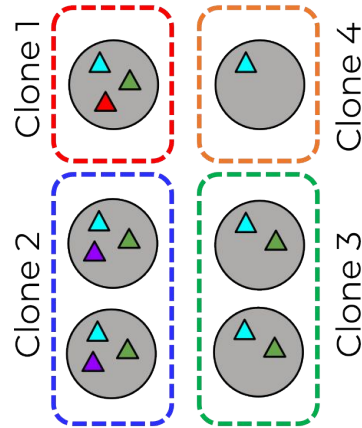


2

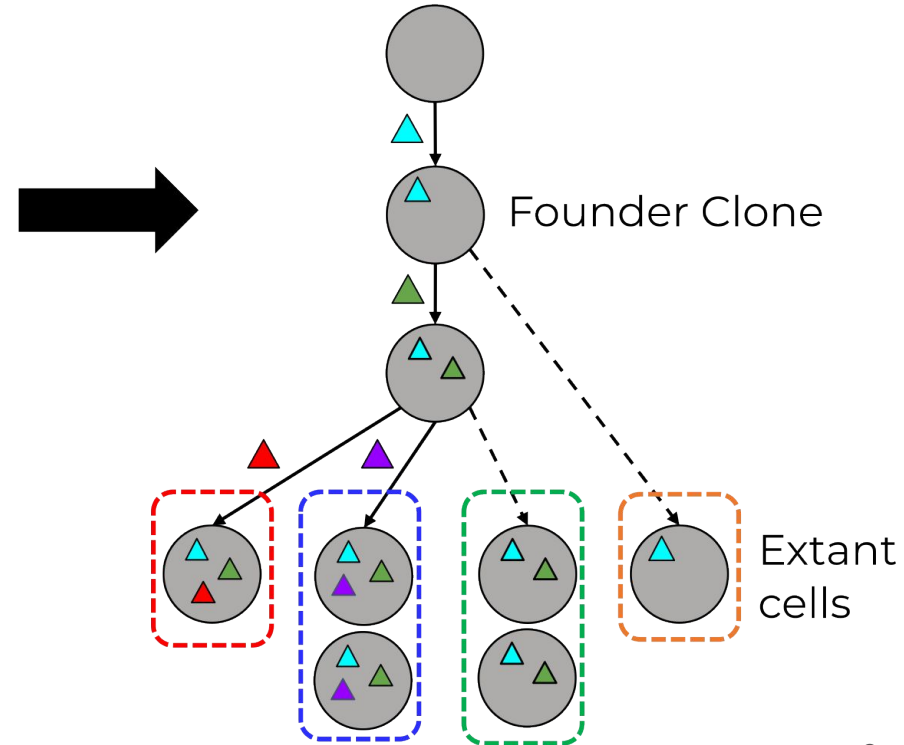


# Reconstructing the evolutionary history of a tumor is a challenging and important open question

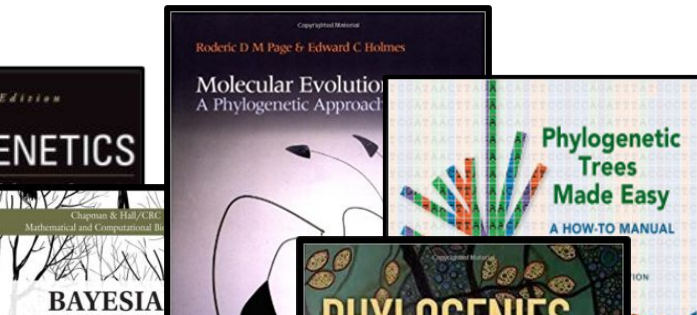
Individual Cells



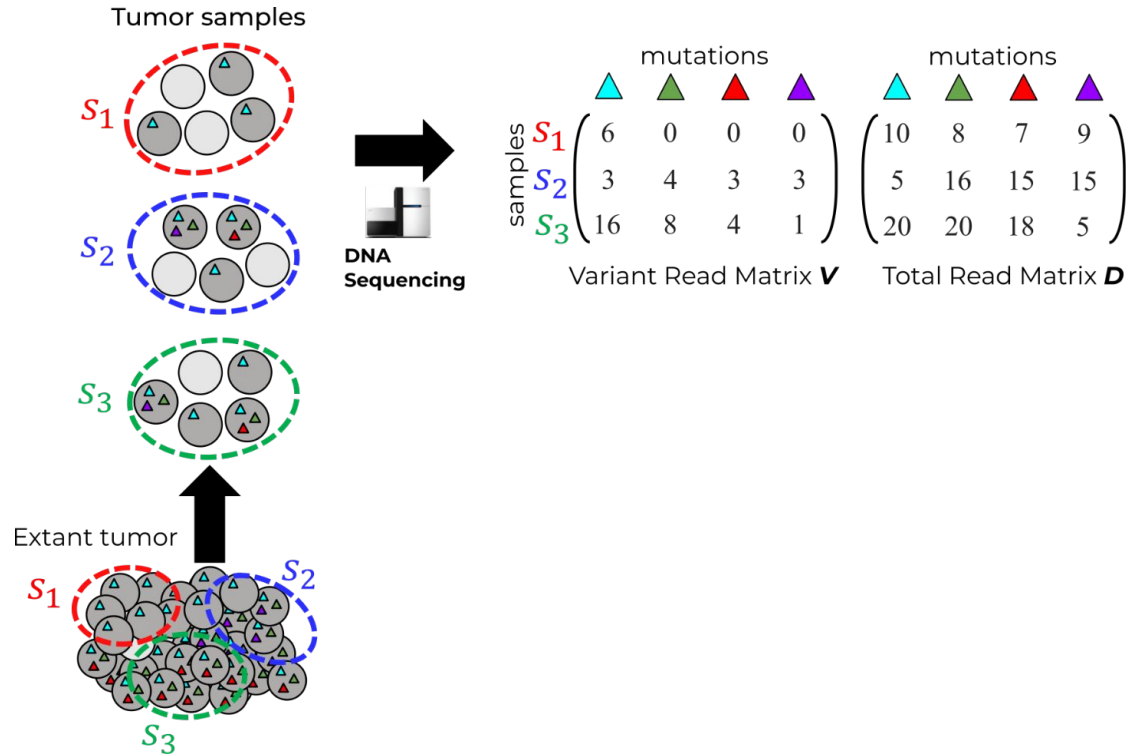
## Tumor Phylogeny



Clone = group of cells with identical genotypes

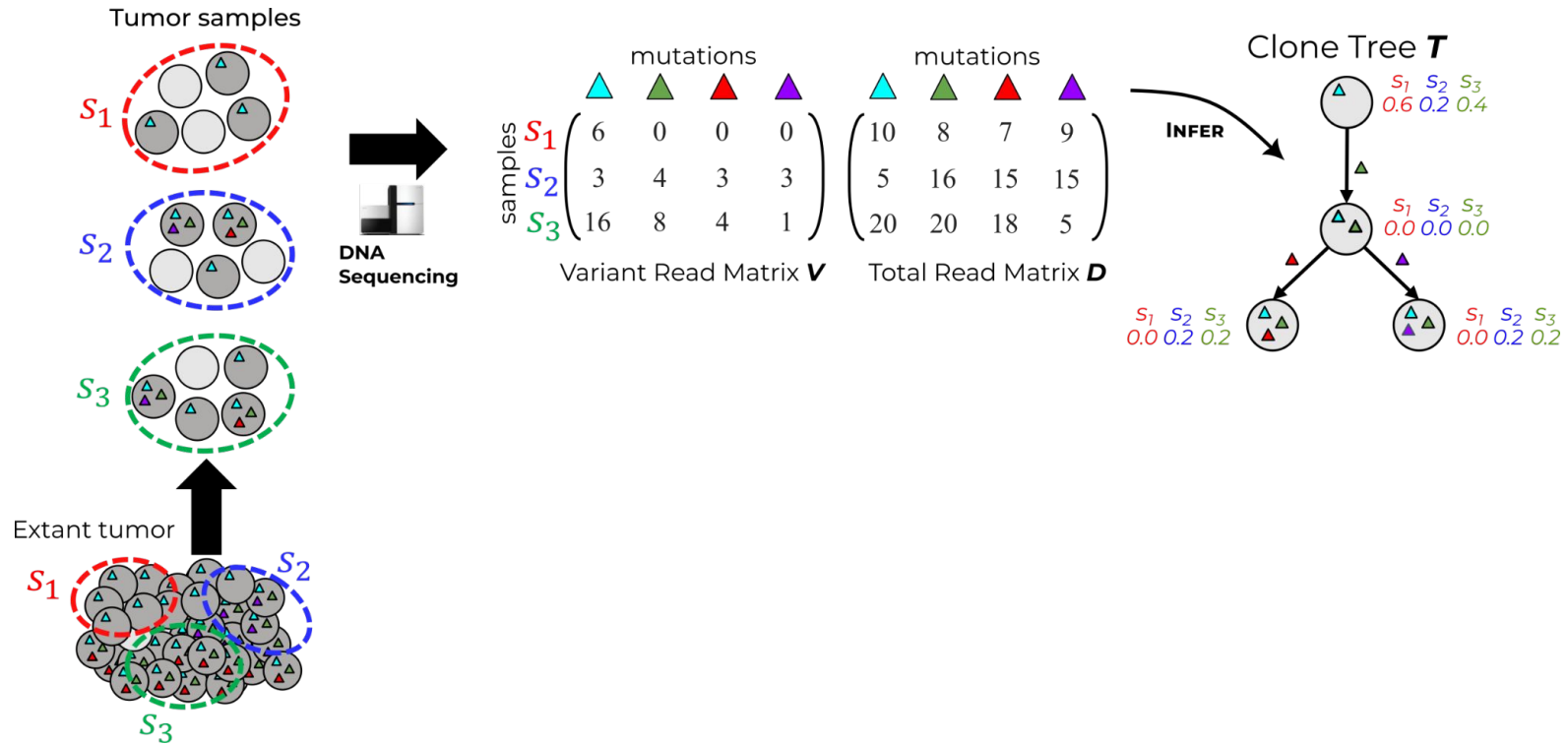


# Bulk DNA sequencing yields a mixture of cells, requiring simultaneous inference of clones and their proportions

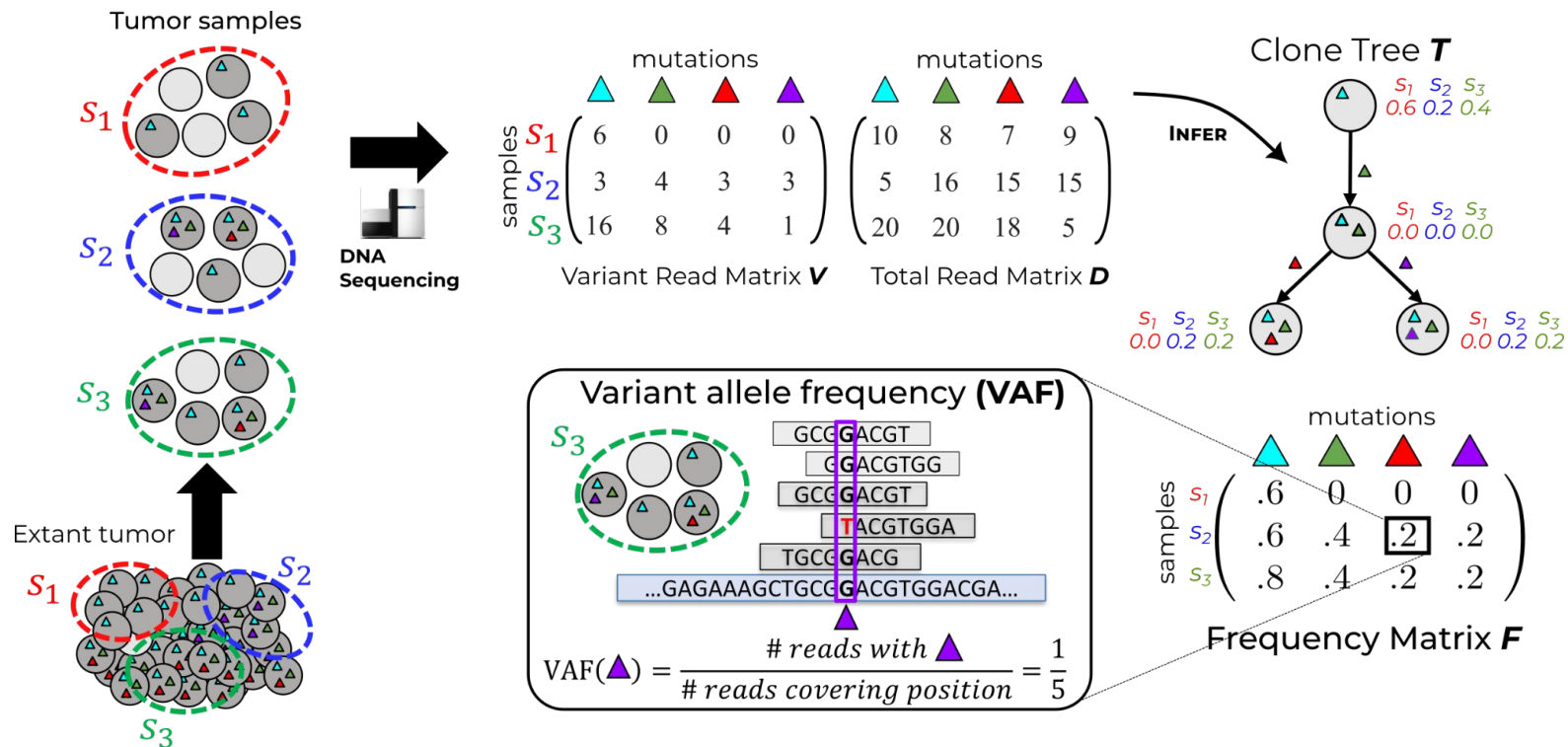


$\frac{3}{2}$

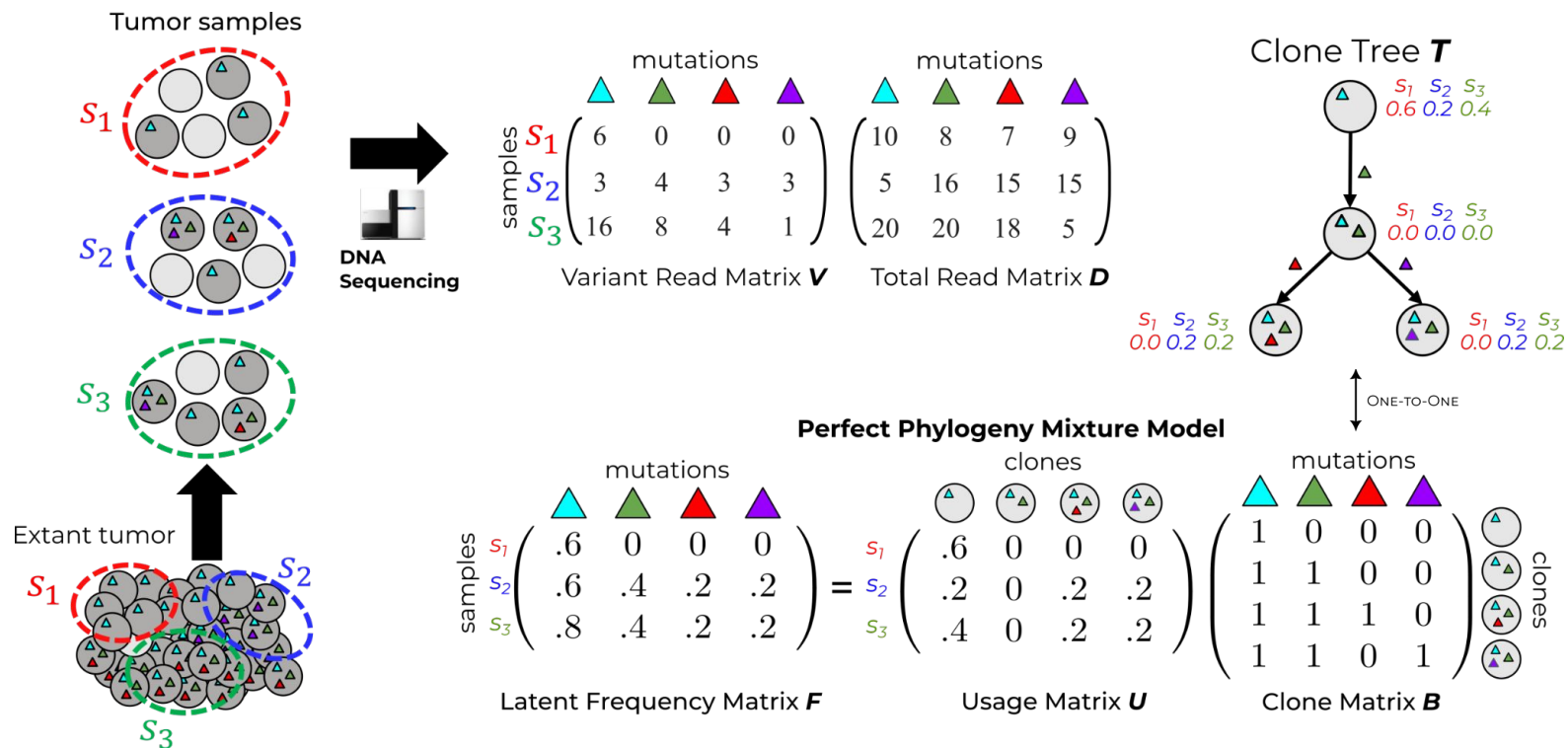
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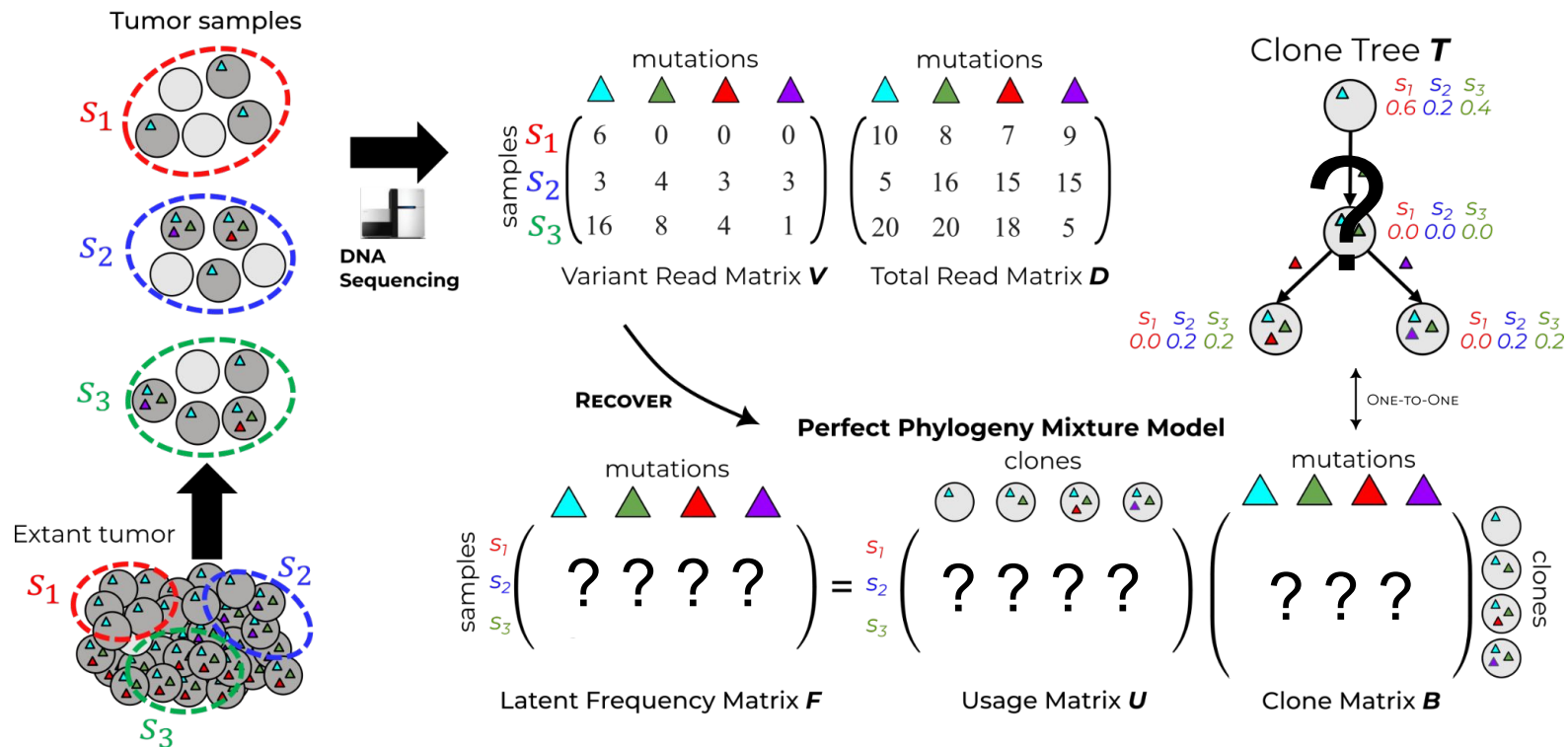


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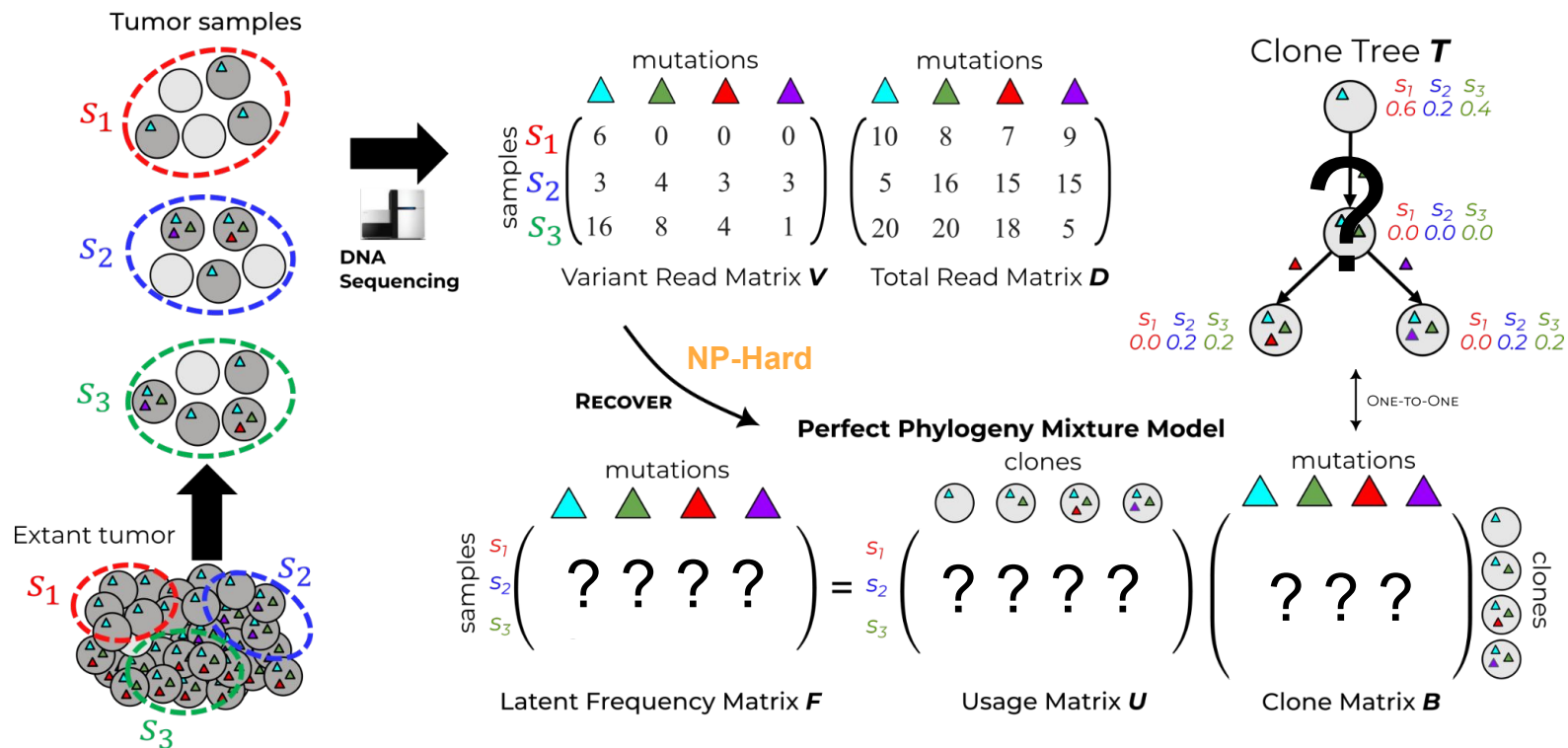
\* This model is implicit or explicit in: **PhyloSub** (Jiao et al., 2014), **PhyloWGS** (Deshwar et al., 2015), **CITUP** (Malikic et al., 2015), **LICHeE** (Popic et al., 2015), **AncesTree** (El-Kebir et al., 2015), **Canopy** (Jiang et al., 2016), **PairTree** (Wintersinger et al., 2022), **Orchard** (Kulman et al., 2023), **fastBE** (Schmidt et al., 2024), ...

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# The perfect phylogeny regression problem





mutations

				
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samples  $S_1$   $S_2$   $S_3$

$$\begin{pmatrix} 6 & 0 & 0 & 0 \\ 3 & 4 & 3 & 3 \\ 16 & 8 & 4 & 1 \end{pmatrix}$$

Variant Read Matrix  $V$

				
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samples  $S_1$   $S_2$   $S_3$

$$\begin{pmatrix} 10 & 8 & 7 & 9 \\ 5 & 16 & 15 & 15 \\ 20 & 20 & 18 & 5 \end{pmatrix}$$

Total Read Matrix  $D$

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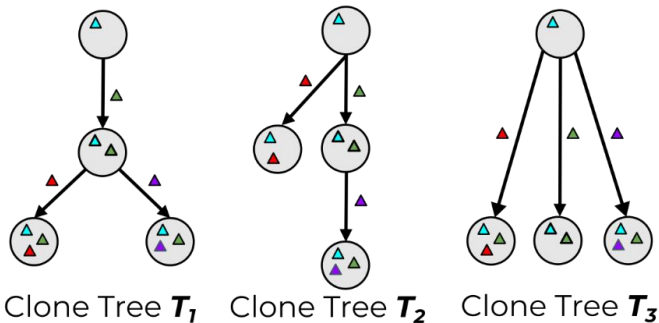
		mutations			
		▲	▲	▲	▲
samples	$S_1$	6	0	0	0
	$S_2$	3	4	3	3
	$S_3$	16	8	4	1

Variant Read Matrix  $\mathbf{V}$

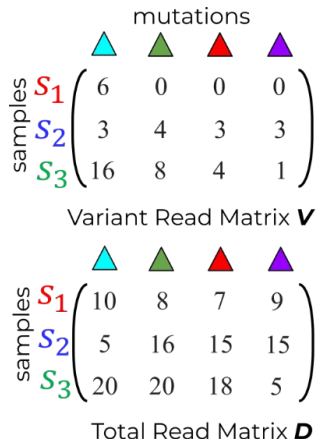
		▲	▲	▲	▲
samples	$S_1$	10	8	7	9
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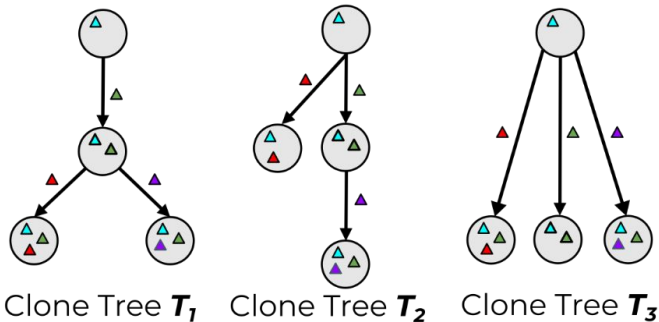
Candidate Tree Set  $\mathcal{T} = \{T_1, T_2, T_3, \dots\}$



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Score & rank trees





SCORES	$L(\mathbf{F}_1   \mathbf{V}, \mathbf{D}) = 0.25$
	$L(\mathbf{F}_2   \mathbf{V}, \mathbf{D}) = 0.73$
	$L(\mathbf{F}_3   \mathbf{V}, \mathbf{D}) = 0.95$
	⋮

by fitting frequencies  $\mathbf{F}$  to trees

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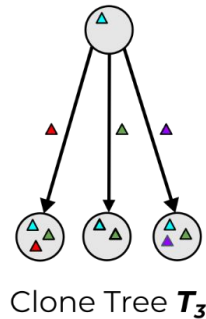
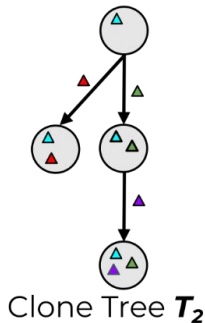
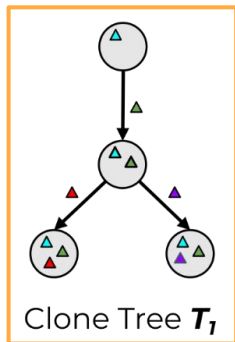
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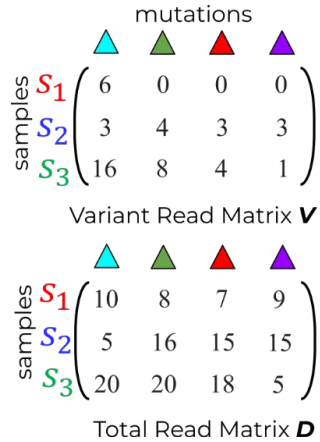
Select best tree(s)

Score & rank trees

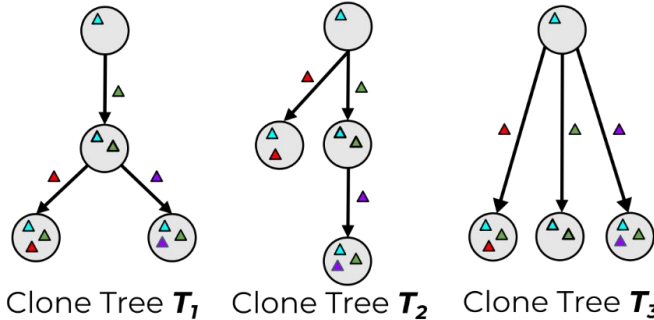
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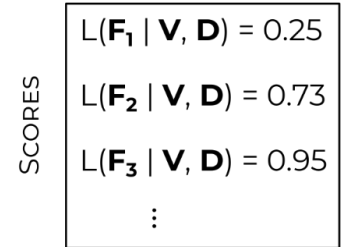
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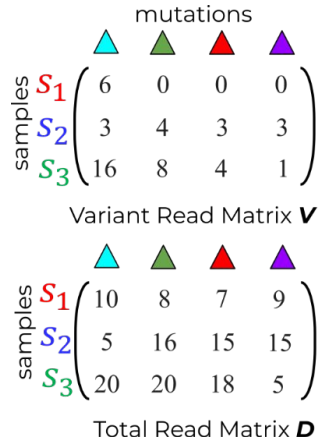
Trees are scored by **repeatedly\*** solving the **perfect phylogeny regression problem**:

**B is fixed**

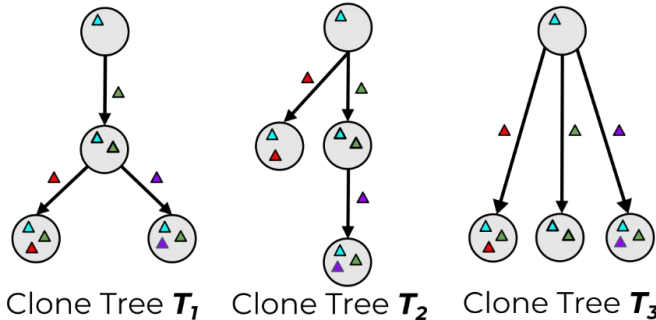
$$(PPR) \quad \min_{\mathbf{F}, \mathbf{U}} \{L(\mathbf{F} | \mathbf{V}, \mathbf{D}) : \mathbf{F} = \mathbf{U}\mathbf{B}, \mathbf{U} \geq 0, \mathbf{U}\mathbf{1} \leq 1\}$$

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Solving the PPR problem is the **key computational bottleneck** in phylogeny inference algorithms.

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# The perfect phylogeny regression problem

For example, when a least-squares loss is used

$$L(\mathbf{F} \mid \mathbf{V}, \mathbf{D}) = \sum_{i=1}^m \sum_{j=1}^n \left( f_{ij} - \frac{v_{ij}}{d_{ij}} \right)^2$$

e.g. as in **CITUP** (Malikic et al., 2015), **PairTree** (Wintersinger et al., 2022), and **Orchard** (Kulman et al., 2023)

the PPR problem is solved in polynomial time using either quadratic programming or specialized solvers.

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**Pitfall #1: Does not model read counts**



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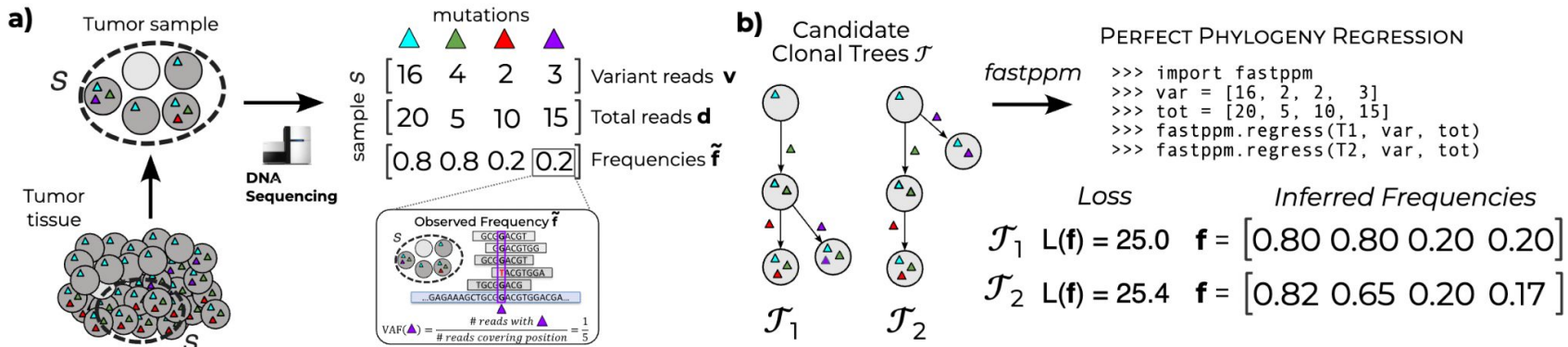
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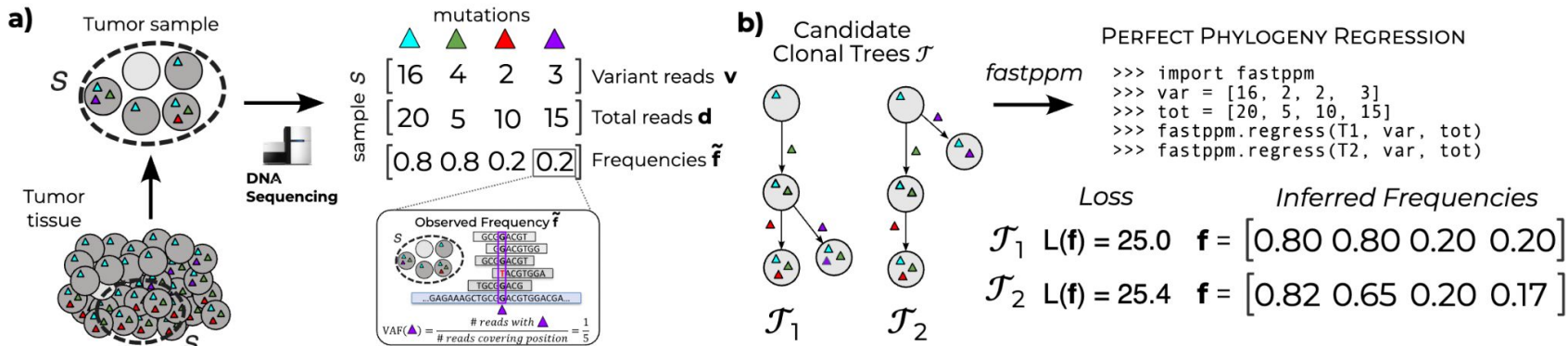
## Pitfall #2: General purpose solvers are slow

# Contributions



We introduce a new approach to the Perfect Phylogeny Regression problem, *fastppm*, using *tree structured dual dynamic programming (TSDDP)*.

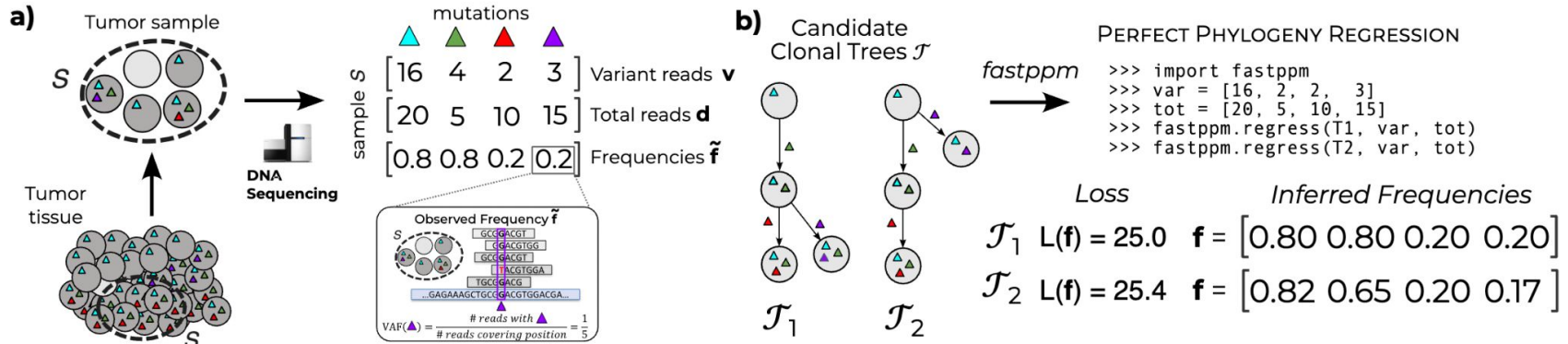
# Contributions



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1. *fastppm* provides **asymptotic** and **empirical** (50-100x speed-up) speedups for the most commonly used  $L_2$  and  $L_1$  loss functions.

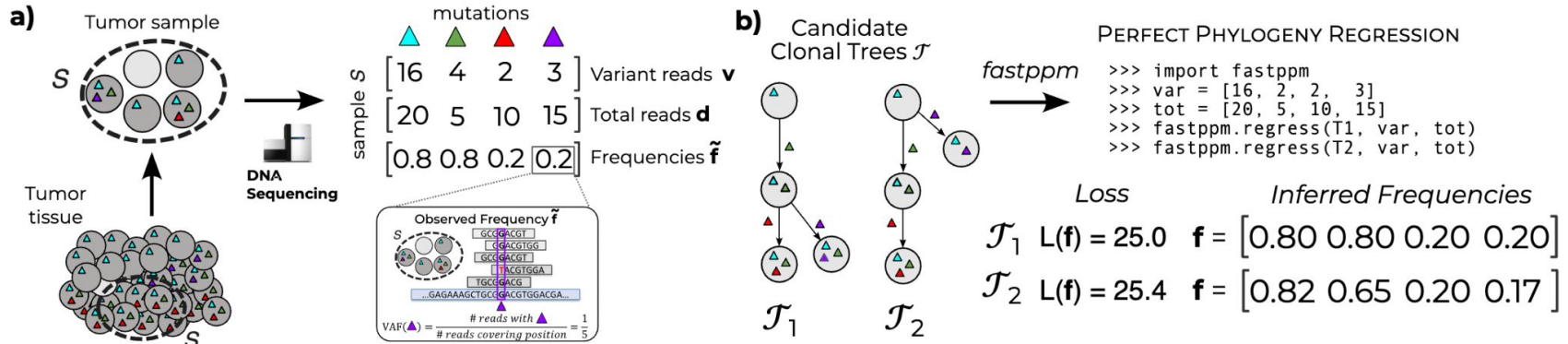
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# Contributions



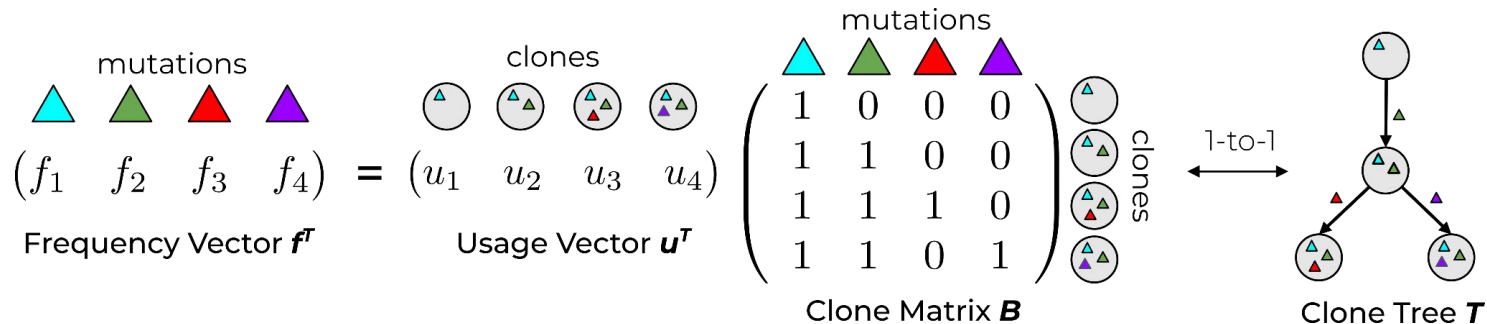
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2. *fastppm* is able to model arbitrary, **convex loss functions**, while maintaining its performance.

On simulated data, replacing existing solvers with *fastppm* yields up to **400x** speed-ups and enables fast + accurate phylogenetic inference from **low-coverage** bulk DNA sequencing data.

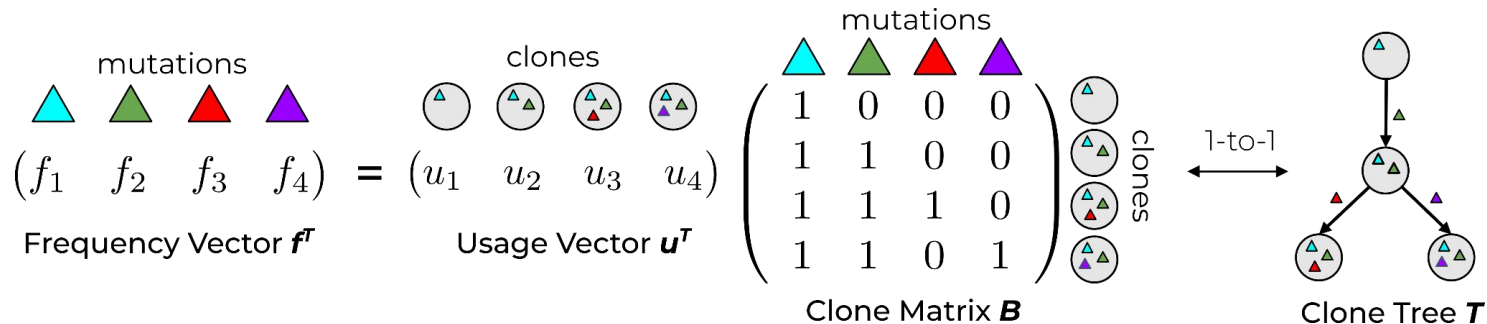
# Tree structured dual dynamic programming (TSDDP)

For simplicity, we study the PPR problem in the case of a single sample:



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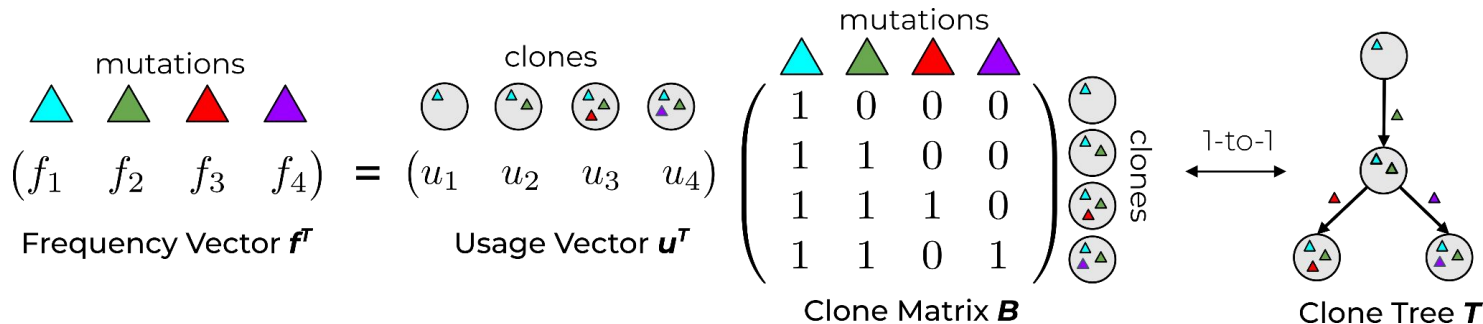
Then, our optimization problem is

$$(PPR) \quad \min_{\mathbf{f}, \mathbf{u} \in \mathbb{R}^n} \left\{ \sum_{i=1}^n L_i(f_i) : \mathbf{f}^T = \mathbf{u}^T \mathbf{B}, \mathbf{u} \geq 0, \mathbf{u}^T \mathbf{1} \leq 1 \right\}.$$



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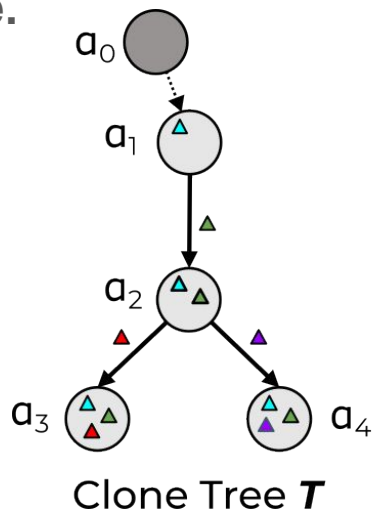
In TSDDP, we first construct the **dual** optimization problem

$$(Dual-PPR) \quad \max_{\boldsymbol{\alpha} \in \mathbb{R}^{n+1}} \left\{ -\alpha_0 + \sum_{i=1}^n h_i(\alpha_i - \alpha_{\pi(i)}) : \boldsymbol{\alpha} \geq 0 \right\},$$

where  $\pi(i)$  is the parent of vertex  $i$  in  $T$  and  $h_i$  is conjugate to  $L_i$ .

# Tree structured dual dynamic programming (TSDDP)

Example.



VARIABLES:

$a_0, a_1, a_2, a_3, a_4$  ON VERTICES OF  $T$

OBJECTIVE:

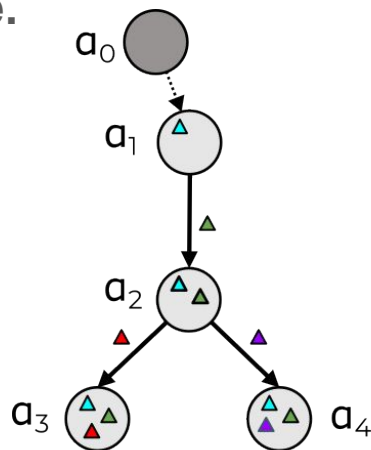
$$h_1(a_1 - a_0) + h_2(a_2 - a_1) + h_3(a_3 - a_2) + h_4(a_4 - a_2)$$

CONSTRAINTS:

$$a_0 \geq 0, a_1 \geq 0, a_2 \geq 0, a_3 \geq 0, a_4 \geq 0$$

# Tree structured dual dynamic programming (TSDDP)

Example.



Clone Tree  $T$

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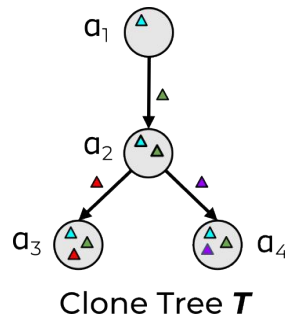
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**Key Idea:** Solve the **dual** problem with a **bottom-up dynamic programming algorithm** over the clone tree  $T$ .

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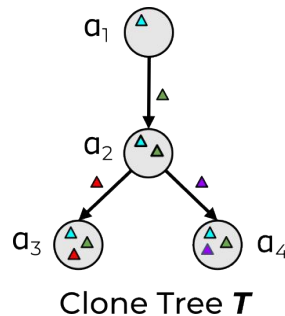
Specifically, we define

$$J_i(\gamma) \triangleq \max_{\alpha \geq 0} \left\{ \sum_{j \in D(i)} h_j(\alpha_j - \alpha_{\pi(j)}) : \alpha_{\pi(i)} = \gamma \right\}$$

which is the **optimal solution to the dual problem for the subtree rooted at vertex  $i$ , provided the dual variable of the parent of vertex  $i$  takes value  $\gamma$ .**

# Tree structured dual dynamic programming (TSDDP)

**Key Idea:** Solve the **dual** problem with a **bottom-up dynamic programming algorithm** over the clone tree  $\mathbf{T}$ .



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which is the **optimal solution to the dual problem for the subtree rooted at vertex  $i$ , provided the dual variable of the parent of vertex  $i$  takes value  $\gamma$** . Then, this function satisfies the recurrence relation

$$J_i(\gamma) = \max_{\alpha_i \geq 0} \left\{ h_i(\alpha_i - \gamma) + \sum_{j \in \delta(i)} J_j(\alpha_i) \right\} \quad \text{(Recurrence Relation)}$$

and TSDDP then computes the functions  $J_i$  in a **bottom-up** fashion.

# Tree structured dual dynamic programming (TSDDP)

For the *weighted* least squares loss, we solve the PPR problem in  $\mathcal{O}(n^{3/2}\log(\log(n)))$  time\* over classes of random trees:

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## Efficient Projection onto the Perfect Phylogeny Model

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**Best known:**  $\mathcal{O}(n^2)$

**Our result:**  $\mathcal{O}(n^{3/2}\log(\log(n)))$

\*  $n$  is the number of clones in the clone tree  $T$ .

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For the *piecewise linear loss* with  $k$  pieces, we solve the PPR problem in  $\mathcal{O}(n\log^2(nk))$  time deterministically:

RESEARCH ARTICLE

A regression based approach to phylogenetic reconstruction from multi-sample bulk DNA sequencing of tumors

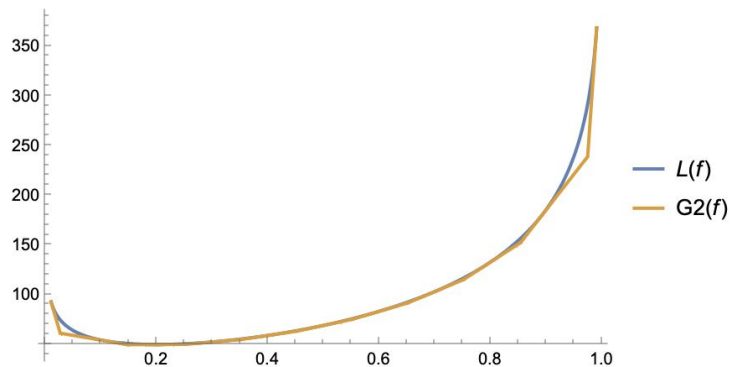
**Best known:**  $\mathcal{O}(n^{3/2})$

**Our result:**  $\mathcal{O}(n\log^2(nk))$

\*  $n$  is the number of clones in the clone tree  $T$ .

# Extensions to arbitrary, convex loss functions

## Approach #1: Piecewise Linear Approximation ( $k$ -PLA and PPLA)

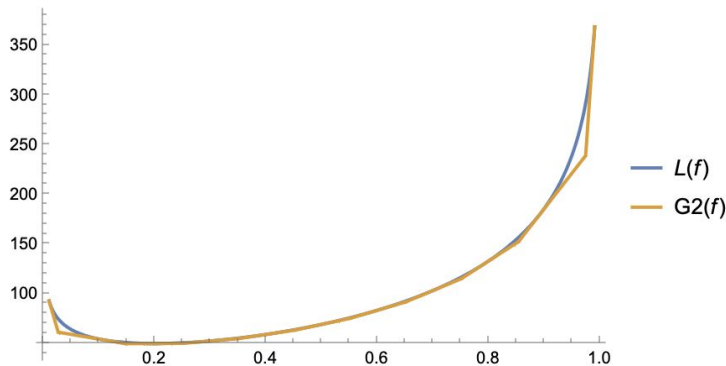


Approximate one-dimensional convex loss function  $L_i(f)$  with piecewise linear approximation using  $k$  pieces found via Taylor series expansion.



# Extensions to arbitrary, convex loss functions

## Approach #1: Piecewise Linear Approximation ( $k$ -PLA and PPLA)



Approximate one-dimensional convex loss function  $L_i(f)$  with piecewise linear approximation using  $k$  pieces found via Taylor series expansion.

## Approach #2: Structured Regression using Alternating Directions Method of Multipliers (ADMM)

ADMM:

$$x^{k+1} := \operatorname{argmin}_x L_\rho(x, z^k, y^k) \quad // \textit{x-minimization}$$

$$z^{k+1} := \operatorname{argmin}_z L_\rho(x^{k+1}, z, y^k) \quad // \textit{z-minimization}$$

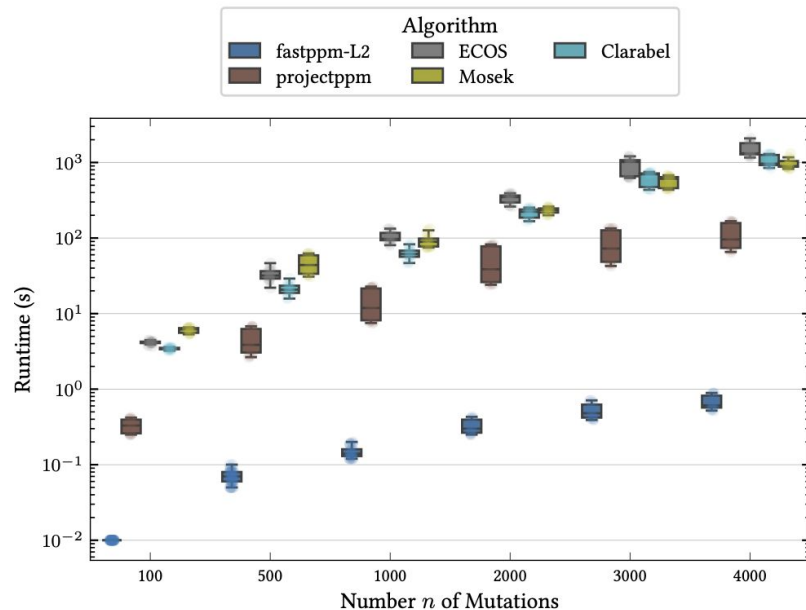
$$y^{k+1} := y^k + \rho(Ax^{k+1} + Bz^{k+1} - c) \quad // \textit{dual update}$$

Using ADMM, we reduce solving the Perfect Phylogeny Regression problem for **arbitrary convex loss functions to a sequence of  $L_2$  subproblems.**

# Fast regression under the perfect phylogeny model

## Results ( $L_2$ / least squares loss):

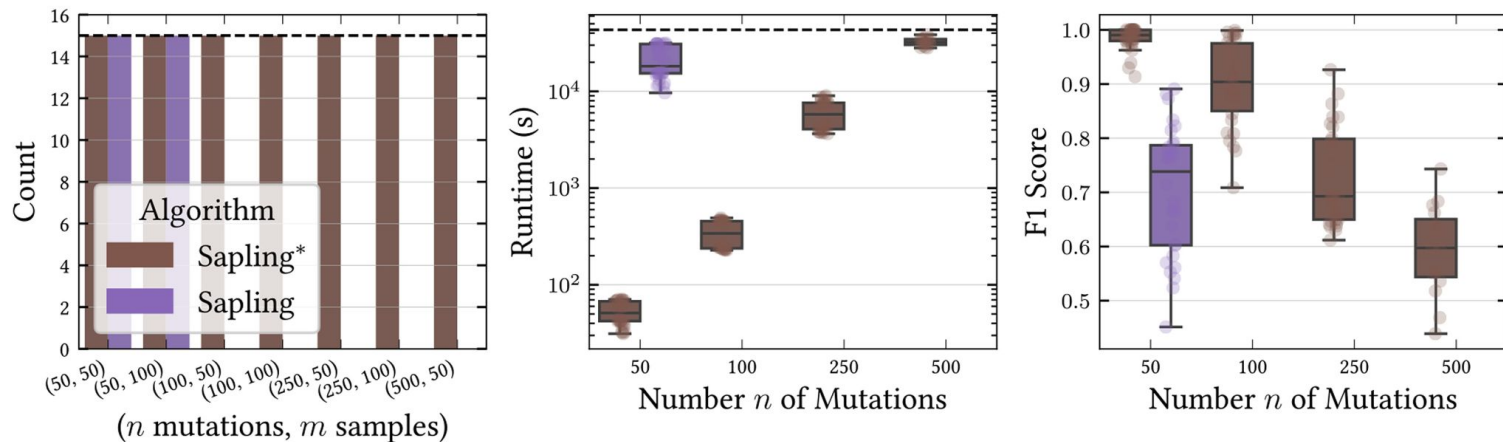
- *fastppm* achieved a 40-125x speed up over the next best performing method *projectppm*.
- All methods achieved the exact same objective value on all instances.
- Blackbox convex optimization solvers were significantly slower than *projectppm* and *fastppm*.
- Excludes model build time which is in practice non-negligible for blackbox solvers.



**Figure:** Runtime of existing methods for the Perfect Phylogeny Regression problem when varying the number of clones/mutations.

# Improving existing phylogeny inference methods with *fastppm*

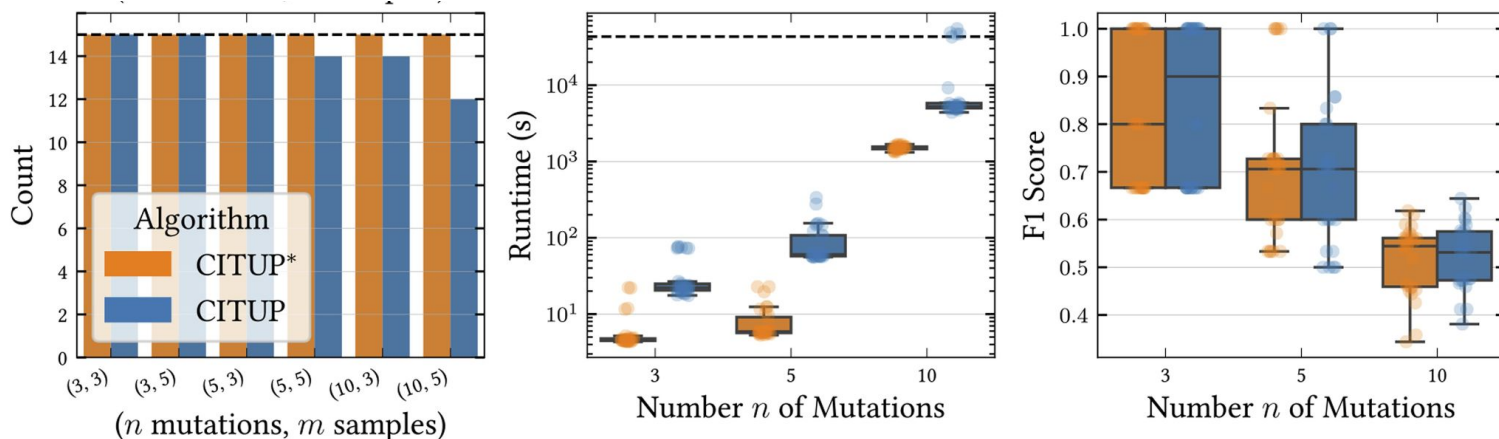
We replaced calls to existing perfect phylogeny regression algorithms in Sapling, CITUP, and Orchard with calls to *fastppm*:



**Figure:** Number of successful instances within a twelve hour time limit, runtime, and accuracy of Sapling compared to Sapling\* on simulated data.

# Improving existing phylogeny inference methods with *fastppm*

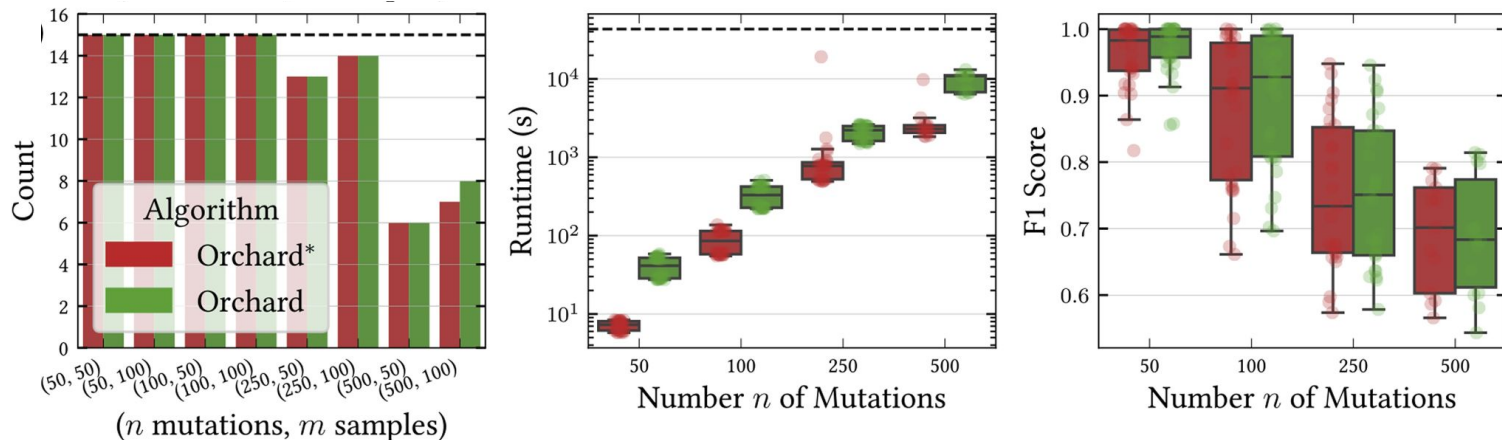
We replaced calls to existing perfect phylogeny regression algorithms in Sapling, CITUP, and Orchard with calls to *fastppm*:



**Figure:** Number of successful instances within a twelve hour time limit, runtime, and accuracy of CITUP compared to CITUP\* on simulated data.

# Improving existing phylogeny inference methods with *fastppm*

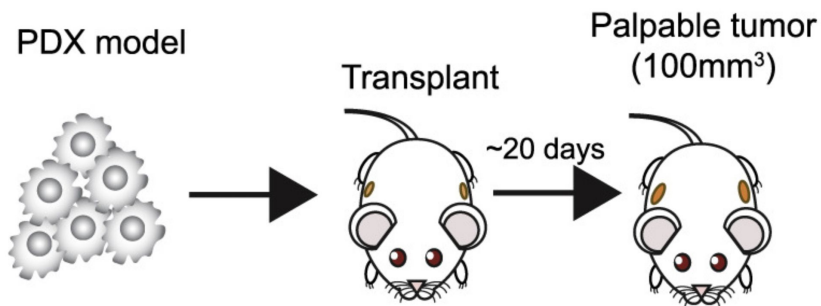
We replaced calls to existing perfect phylogeny regression algorithms in Sapling, CITUP, and Orchard with calls to *fastppm*:



**Figure:** Number of successful instances within a twelve hour time limit, runtime, and accuracy of Orchard compared to Orchard\* on simulated data.

# *fastppm* improves frequency estimation in low coverage settings

Downsampled reads from a patient derived xenograft (POP66) is a mouse model of colorectal cancer ( $n = 64$  mutations,  $m = 8$  samples, 50x coverage):



Data from: (Rehman et al. 2021)

Method	Metric	Objective
Orchard*	$\ \tilde{\mathbf{F}} - \hat{\mathbf{F}}\ _F^2$	3.092
Orchard	$\ \tilde{\mathbf{F}} - \hat{\mathbf{F}}\ _F^2$	2.448
Sapling*	$\ \tilde{\mathbf{F}} - \hat{\mathbf{F}}\ _F^2$	<b>2.181</b>
Orchard*	$-\log \mathbb{P}(\mathbf{V} \mid \mathcal{T}, \hat{\mathbf{F}}, \mathbf{D})$	10790.9
Orchard	$-\log \mathbb{P}(\mathbf{V} \mid \mathcal{T}, \hat{\mathbf{F}}, \mathbf{D})$	10793.5
Sapling*	$-\log \mathbb{P}(\mathbf{V} \mid \mathcal{T}, \hat{\mathbf{F}}, \mathbf{D})$	<b>10720.6</b>

We applied Orchard, Orchard\*, which use the  $L_2$  loss, to Sapling\*, which uses the binomial loss, to recover the clonal tree and mutation frequencies.

# Thank You

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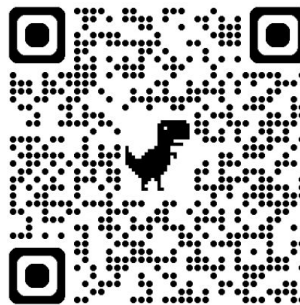
Viola Chen

Julian Gold

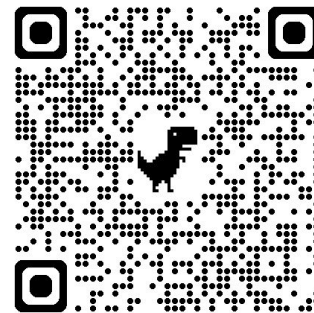


Mohammed El-Kebir

## The Raphael Lab



*fastppm* is implemented in C++  
and is available on GitHub



Manuscript is accessible  
through *Bioinformatics*

SCRATCH



However, existing methods for the *PPR* problem are flawed

1. ***Do not directly*** model the read count data, which e.g. hinders analysis of ***low-coverage DNA sequencing***
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## Example 1.

State-of-the-art phylogeny inference pipelines *CITUP*, *AncesTree*, *CALDER*, *Pairtree*, *Orchard*, and *fastBE* use the following two loss functions:

$$L_1(\mathbf{F}, \mathbf{V}, \mathbf{D}) = \sum_{i=1}^m \sum_{j=1}^n |f_{ij} - \tilde{f}_{ij}| \text{ where } \tilde{f}_{ij} = v_{ij}/d_{ij} \quad \leftarrow \text{Observed frequency (VAF)}$$
$$L_2(\mathbf{F}, \mathbf{V}, \mathbf{D}) = \sum_{i=1}^m \sum_{j=1}^n w_{ij} (f_{ij} - \tilde{f}_{ij})^2 \text{ where } \tilde{f}_{ij} = v_{ij}/d_{ij}, w_{ij} \geq 0$$

which do not directly model the read count data, instead collapsing it to a frequency.

However, existing methods for the *PPR* problem are flawed

1. ***Do not directly*** model the read count data, which e.g. hinders analysis of ***low-coverage DNA sequencing***
  2. Employ ***slow***, black-box convex optimization software which ***do not exploit the structure of the regression problem***
- 

### Example 2.

In contrast, phylogeny inference pipelines (e.g. Sapling, PhyloWGS) which model observations using the probabilistic read-count model  $v_{ij} \sim \text{Binomial}(f_{ij}, d_{ij})$ , e.g.,

$$L_{\text{Bin}}(\mathbf{F}, \mathbf{V}, \mathbf{D}) = - \sum_{i=1}^m \sum_{j=1}^n [v_{ij} \log f_{ij} + (d_{ij} - v_{ij}) \log(1 - f_{ij})].$$

must resort to **blackbox convex optimization software** which is prohibitively slow.

# Tree structured dual dynamic programming (TSDDP)

Then, we solve the **dual** problem with a **bottom-up dynamic programming algorithm over T**.

- (i) Fix a representation  $\mathcal{R}(J_i)$  for each  $J_i$ .
- (ii) Compute the representation  $\mathcal{R}(J_i)$  at the leaf nodes.
- (iii) Compute the representation  $\mathcal{R}(J_i)$  at a node  $i$  provided the representations  $\mathcal{R}(J_j)$  at all children  $j \in \delta(i)$ .
- (iv) Solve the one-dimensional optimization problem  $\max_{\alpha_0 \geq 0} \{-\alpha_0 + J_r(\alpha_0)\}$  using the representation of the root node  $\mathcal{R}(J_r)$ .

## The TSDDP Algorithm